Universality class for extinction-survival phase transition in one dimension

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In one-dimension irreversibly interacting-particle systems exhibiting extinction-survival phase transitions, such as the contact process and branching annihilation random walk, are analyzed using a space-time renormalization-group method. Strong positive evidence is shown for the conjecture that all "reasonably" defined interacting-particle systems with nearest-neighbor interactions belong to the same universality class of directed percolation.

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Topics from diverse fields of sciences, such as catalytic reactions [1], earthquakes [2], granular dynamics [3], and traffic jams [4], are attracting the interest of many statistical physicists recently as examples of irreversible dynamic systems showing phase transitions. The most interesting point is that their critical behavior is often very similar to that of thermal equilibrium systems although they are far from equilibrium. In order to elucidate the essential mechanism of dynamic phase transitions, it is worthwhile to investigate minimal mathematical models.

Among many models showing dynamic phase transitions, the contact process (CP) [5] and the branching annihilation random walk (BARW) [6,7] are considered to be typical minimal models. Both models are defined on a lattice where particles are created or annihilated randomly.

In the case of CP, a randomly chosen particle either creates a particle on an empty neighbor site or it is annihilated randomly. If the rate of annihilation dominates the creation rate, all particles die out. On the other hand, at large creation rates particles can survive forever. This is a typical dynamic phase transition between an extinction phase and a survival phase. It is known that CP in d dimensions belongs to the same universality as directed percolation (DP) in $d+1$ dimensions [8].

BARW is defined in a similar way. A randomly chosen particle creates a particle on a neighbor site or walks randomly to a neighbor site. When two particles try to share a site they both annihilate simultaneously (pair annihilation). The extinction-survival transition can also occur in the case of BARW. For a large creation rate we have a survival state as in the case of CP. However, the existence of the extinction state is rather delicate for BARW. Since annihilation takes place only at collisions, the occurrence frequency is proportional to the square of the particle density in a mean-field sense while the birth frequency is a linear function of the density. This implies that the annihilation process is negligible for a low-density state meaning that we always have a survival phase. This mean-field picture is valid only for space dimensions higher than 2. Numerical evidence [7] and a rigorous proof for one-dimension shows [6] that there exists a nontrivial extinction state in one-dimension. The particle density ρ is known to follow the familiar critical behavior.

$$\rho \propto |p-p_c|^{\beta} , \tag{1}$$

where p is the rate of choosing a random walk and β and p_c denote the critical exponent and critical point, respectively.

While first estimations of β [7] produced a value a little larger than the one for CP, $b=0.2769(2)$ [9], Jensen [10] shows that the values of dynamical exponents, related to time behavior at the critical point, are very close to DP values, strongly indicating that BARW and CP are in the same universality class, in spite of very different dynamic rules.

In this paper, using space-time renormalization, we show strong theoretical evidence for this conjecture in a much more general sense. All "reasonably" defined interacting particle systems with nearest-neighbor interactions in one-dimension converge to CP, i.e., they all belong to the DP universality class in a macroscopic scale.

A general case of an interacting-particle system with nearest-neighbor interactions is defined by repeating the following dynamics: Choose a nearest-neighbor pair of sites at random and change the pair randomly with a specified rate. There are four possible configurations for a pair, 00, 01, 10, and 11, where 0 stands for an empty site and 1 for an occupied one, so that the full information on the transition rates is given by a transition matrix. Assuming the right-left symmetry and limiting our consideration to the models with the extinction state as an adsorption point, the transition matrix M for the four configurations is given as

$$M=\begin{pmatrix} 1 & 0 & 0 & 0 \\ a & 1-a-b-c & b & c \\ a & b & 1-a-b-c & c \\ d & e & e & 1-d-2e \end{pmatrix} . \tag{2}$$

Here, we introduce five parameters which denote the occurrence probabilities of the following transitions: a—spontaneous annihilation, $01\to 00$, or $10\to 00$; b—random walk, $01\to 10$, or $10\to 01$; c—branching, $01\to 11$, or $10\to 11$; d—pair annihilation, $11\to 00$; e—single annihilation, $11\to 10$, or $11\to 01$. As special cases

CP and BARW are characterized by a one parameter as

$$\text{(CP)}\quad b=0\ ,\quad c=1-a\ ,\quad d=0\ ,\quad \text{and } e=a/2\ ,$$

$$\text{(BARW)}\quad a=0\ ,\quad c=1-b\ ,\quad d=b\ ,\ \text{and } e=(1-b)/2\ .$$

Now we introduce the idea of renormalization-group method for this general transition matrix. Let us replace every two sites of the original lattice by one supersite of a coarse-grained lattice. A supersite is considered to be occupied if at least one of the two corresponding sites of the original lattice are occupied; it is empty if (and only if) both of the corresponding original sites are empty. As for the time steps, we consider four steps in the original system as 1 step in the coarse-grained dynamics (see Fig. 1). Attempts to vary the number of steps corresponding to one coarse-grained step showed that in the case of five steps the results are almost the same, while for three steps the convergence is much slower, though qualitatively the behavior is very similar.

Every event in the original space has a corresponding event on the superlattice, which defines an interacting-particle problem in the coarse-grained space. Although the original process can be a "pure" CP or BARW, the corresponding process in the coarse-grained space will generally be a "mixed" process.

In order to determine the coarse-grained transition matrix we count up all possible events in an original lattice of four sites and four time steps. To estimate the weights of different initial configurations giving the same state on the superlattice (for example, for the coarse-grained pair, 10, we have three configurations, 1100, 1000, and 0100). We assume a kind of stationary condition for the configuration probability $p(\cdot)$ with the following boundary conditions (mean-field method);

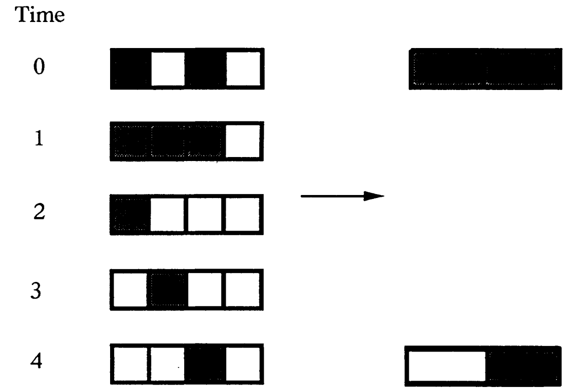

FIG. 1. Coarse graining of the dynamics in the space time.

$$p(\eta(0),\eta(1),\eta(2),\eta(3),\eta(4),t)=\frac{p(\eta(0),\eta(1),\eta(2),\eta(3),t)p(\eta(1),\eta(2),\eta(3),\eta(4),t)}{p(\eta(1),\eta(2),\eta(3),0,t)+p(\eta(1),\eta(2),\eta(3),1,t)}\ , \tag{3}$$

where $\eta(x)$ takes the value 0 or 1, representing a vacant site and a particle on a site. These straightforward but troublesome computations are done by using algebraic calculations on a workstation-type computer.

Figure 2(a) shows the values of the renormalization parameters starting from BARW parameter values with $b=0.103$ as functions of the number of renormalizations, n. In this case the branching probability c grows with n and converges to 1, while the rest of the parameters converge to zero. Figure 2(b) shows the same plot starting with a slightly larger value $b=0.105$. Here, parameters b, c, and d decrease with n, while a and e, corresponding to annihilation processes, increase in value. The change of the stationary particle density in the renormalized system is plotted as a function of b in Fig. 3, which clearly shows the existence of a phase transition at $b=0.1048$.

In the general case [Eq. (2)] we find that in the macroscopic limit the values of b and d always become much smaller than other parameters a, c, and e. This result is intuitively recognized as follows. Assume that we renormalize L sites as a supersite and L time steps into one superstep. A random walker needs about L^2 time steps to move across L sites, so for larger L the contribution of random walk is less effective. A pair annihilation of coarse-grained particles means that all particles in the original $2L$ sites vanish in finite time steps. This occurrence is obviously less likely for larger L.

Since b and d vanish in the macroscopic limit, the macroscopic dynamics can be regarded as CP if the ratio e/a converges to $\frac{1}{2}$ as we repeat the renormalization. As is

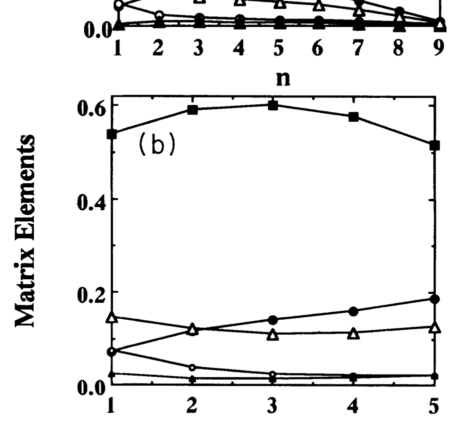

FIG. 2. (a) Change of transition-probability matrix elements in a survival state, where n denotes the number of renormalization. a: solid circles; b: open circles; c: solid squares; d: solid triangles; and e: open triangles. (b) Change of transition probability matrix elements in an extinction state. The notations are the same as in (a).

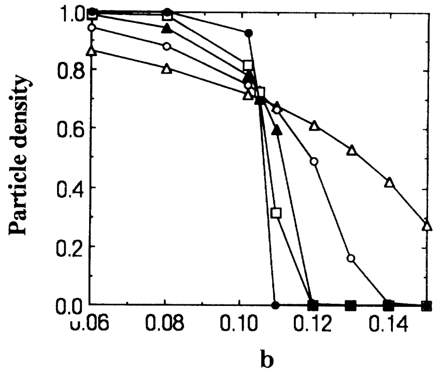

FIG. 3. Stationary particle density as a function of b. The phase transition becomes evident as the number of renormalizations increases. Open triangles ($n=3$), open circles ($n=4$), solid triangles ($n=5$), open squares ($n=6$), and solid circles ($n=7$).

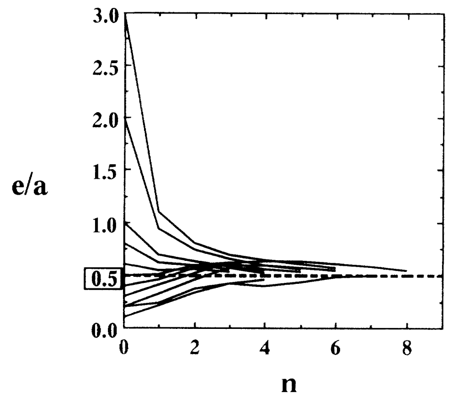

FIG. 4. Change of the ratio e/a by renormalization for various combinations of a, c, and e with $b=d=0$. For different curves a is in the interval from 0.1 to 0.2, while e takes values between 0.02 and 0.3.

shown in Fig. 4, the ratios e/a starting from several different combinations of a, c, and e actually gather around $\frac{1}{2}$ as we repeat the renormalization procedure. This result shows that all systems covered by Eq. (2) can be regarded as CP in the macroscopic limit, and thus belong to the same universality class as DP [8].

Summarizing the results, we have introduced a general representation of interacting-particle systems in one dimension, including CP and BARW. By directly coarse graining the space time we obtained the renormalized transition rates and found that any transition matrix converges to that of CP in the macroscopic limit. The existence of such a huge universality class in the irreversible dynamic critical phenomena may encourage the study of universality in other dynamic systems.

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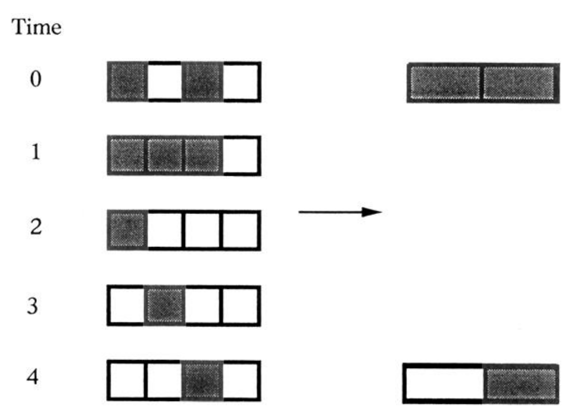

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